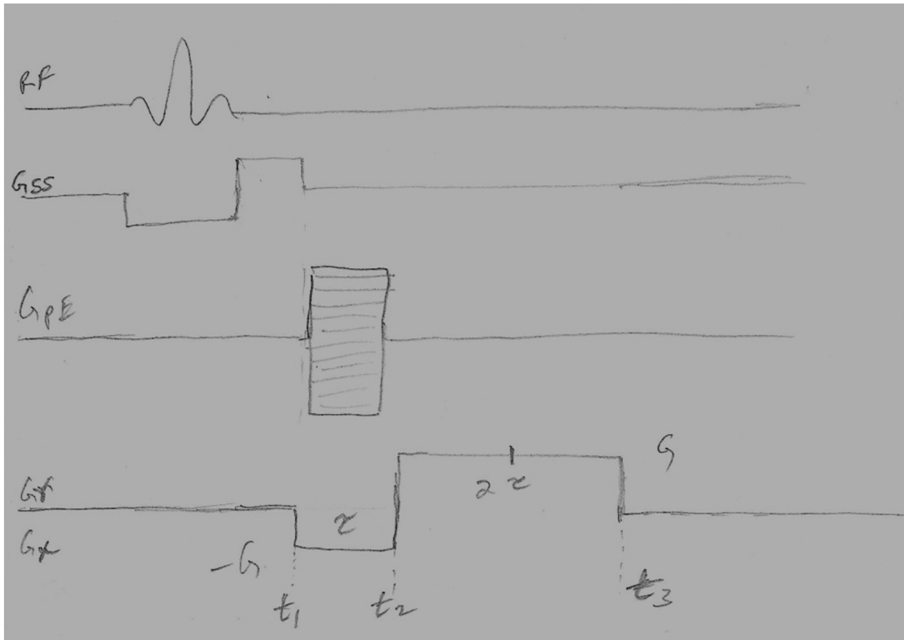
Effects of Spin Phase from Motion along the Read Direction



Flow along read direction only

$$x(t) = x_0 + v_x t$$

$$\phi = \gamma \int G_x(t)\, x \, dt \qquad [\text{No flow case}]$$

$$\phi = \gamma \int G_x(t)\, x(t)\, dt$$

$$= \gamma \int G_x(t)\,(x_0 + v_x t)\, dt$$

$$= \gamma \left[\int_{t_1}^{t_2} -G_x (x_0 + v_x t)\, dt + \int_{t_2}^{t} G_x (x_0 + v_x t)\, dt \right] \quad t_2 < t < t_3$$

$$= \gamma \left[-G_x x_0 t \Big|_{t_1}^{t_2} + \left(-G_x v_x \frac{t^2}{2} \right) \Big|_{t_1}^{t_2} + G_x x_0 t \Big|_{t_2}^{t} + \left(G_x v_x t^2/2 \right) \Big|_{t_2}^{t} \right]$$

$$= \gamma \left[-G_x x_0 (t_2 - t_1) - \frac{G_x v_x (t_2^2 - t_1^2)}{2} + G_x x_0 (t - t_2) + \frac{G_x v_x (t^2 - t_2^2)}{2} \right]$$

$$= \gamma\left[-G_x x_0 (t_2 - t_1) - \frac{G_x V_x (t_2^2 - t_1^2)}{2} \right.$$

$$\left. + G_x x_0 (t - t_2) + \frac{G_x V_x (t^2 - t_2^2)}{2} \right]$$

writing this

Relative to TE $\rightarrow$ $t_1 + 2\tau$, coordinate change

$$t' = t - (t_1 + 2\tau)$$

$$= \gamma\left[-G_x x_0 \tau - \frac{G_x V_x (t_2^2 - t_1^2)}{2} \right.$$

$$+ G_x x_0 (t' + t_1 + 2\tau - t_2)$$

$$\left. + \frac{G_x V_x}{2} \left(t'^2 + t_1^2 + 4\tau^2 + 4\tau t_1 + 2t' t_1 + 4t'\tau - t_2^2 \right) \right]$$

$$= \gamma\Big[-G_x x_0 \tau - \frac{G_x V_x}{2}(t_2^2 - t_1^2) + G_x x_0 (t' - \tau + 2\tau)$$

$$+ \frac{G_x V_x}{2}\left[t_1'^2 - (t_2^2 - t_1^2) + 4\tau(t_1 + \tau) + 2t'(t_1 + 2\tau) \right]\Big]$$

$$= \gamma\Big[G_x x_0 t' - G_x V_x (t_2^2 - t_1^2) + \frac{G_x V_x}{2} t'^2$$

$$+ G_x V_x \left[2\tau(t_1 + \tau) + t'(t_1 + 2\tau) \right]\Big]$$

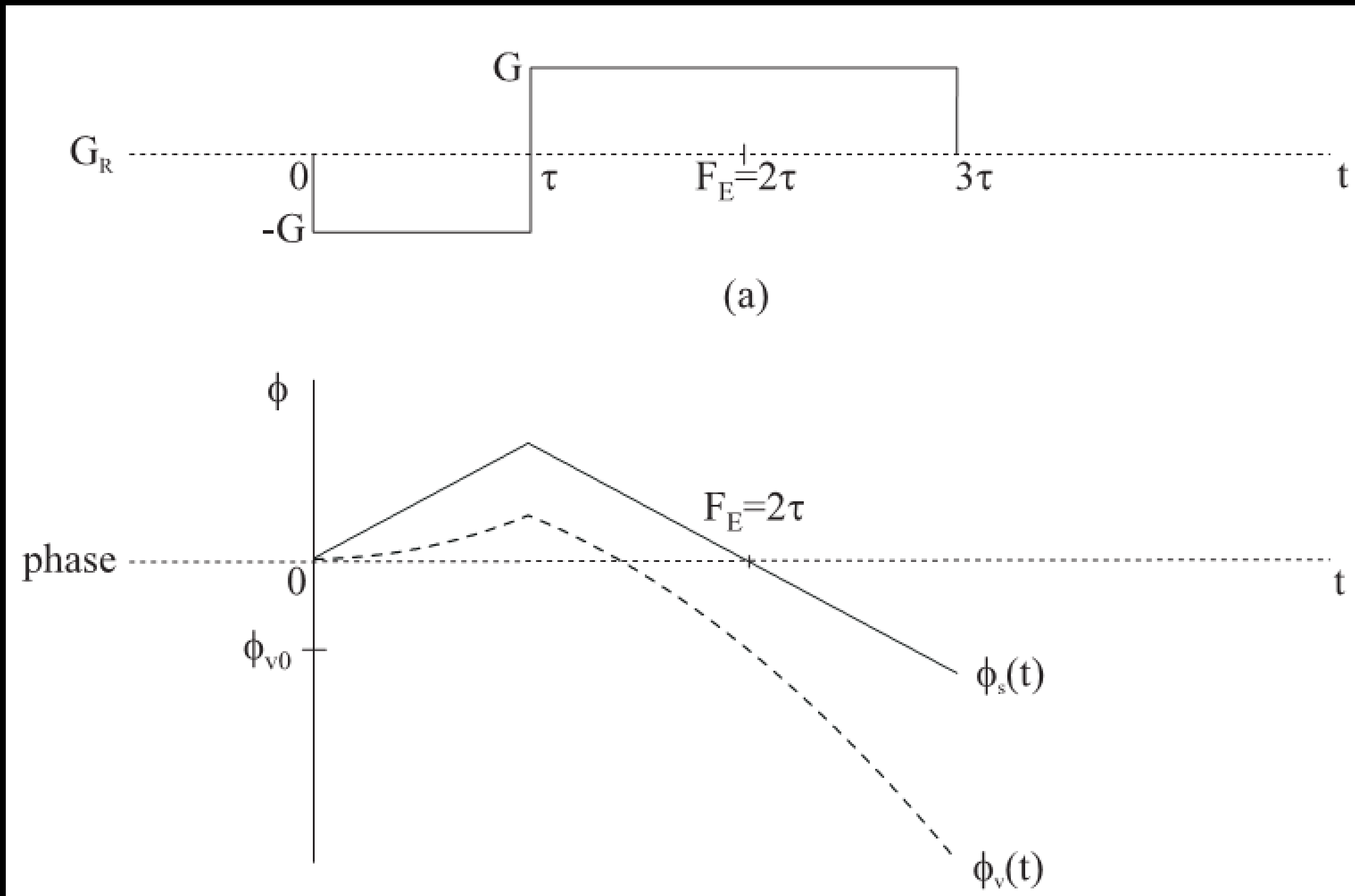
$t_1 = 0$ (origin shift)

$t_2 = \tau$

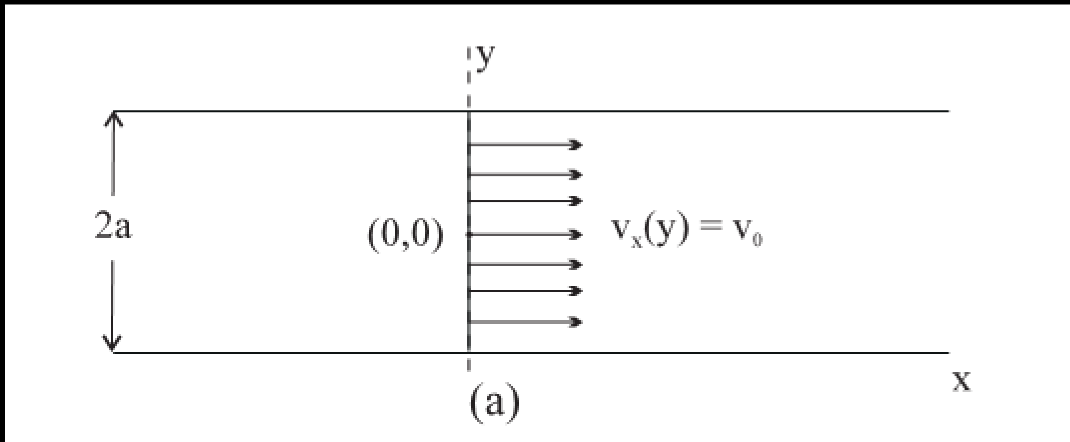
$$\Rightarrow \gamma\left[G_x x_0 t' - G_x V_x \tau^2 + G_x V_x t'^2 + G_x V_x 2\tau^2 + G_x V_x t' 2\tau \right]$$

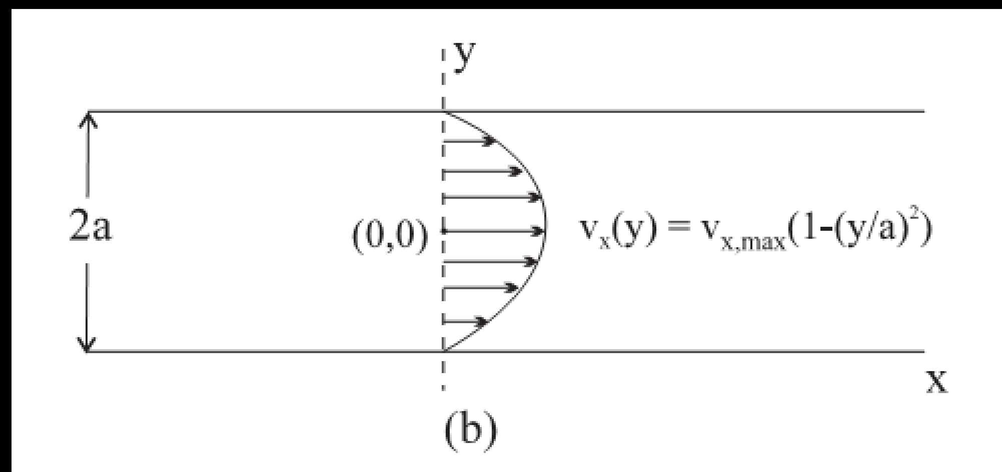
$$\boxed{\phi = \underline{\gamma G_x t'(x_0 + V_x 2\tau + V_x t')} + \underline{\gamma G_x V_x \tau^2}}$$

Additional phase terms



G
G_R
0
τ
$F_E=2\tau$
3τ
t
-G
(a)
ϕ
$F_E=2\tau$
phase
0
t
ϕ_{v0}
$\phi_s(t)$
$\phi_v(t)$

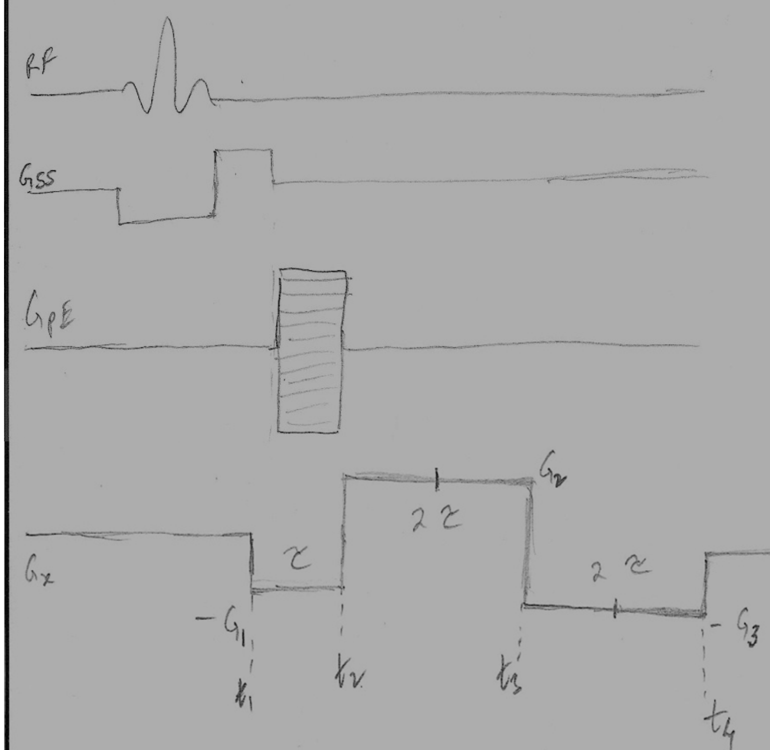




$$v_x(y) = v_{x,max}\left(1 - \left(\frac{y}{a}\right)^2\right)$$

$$v_x(y) = v_x(x_0, y_0) + \alpha(y - y_0)$$

$$\begin{aligned}\hat{\rho}_v(x_0, y_0) &= \frac{\rho_0(x_0, y_0)}{\Delta y} \int_{y_0-\Delta y/2}^{y_0+\Delta y/2} dy\, e^{-i\gamma G v_x(x_0,y_0)\tau^2} e^{-i\gamma G\alpha(y-y_0)\tau^2} \\ &= \rho_0(x_0, y_0) e^{-i\gamma G v_x(x_0,y_0)\tau^2} \operatorname{sinc}(\gamma\alpha G\tau^2 \Delta y/2)\end{aligned}$$



$$\phi = \gamma\left[\int G_x(t)\,(x_0 + v_x t)\,dt\right]$$

$$= \gamma\left[\int_{t_1}^{t_2} -G_1 (x_0 + v_x t)\,dt + \int_{t_2}^{t_3} G_2 (x_0 + v_x t)\,dt + \int_{t_3}^{t} (-G_3)(x_0 + v_x t)\,dt\right]$$

$$= \gamma\Big[-G_1 x_0 (t_2 - t_1) - \frac{G_1 v_x}{2}(t_2^2 - t_1^2)$$

$$+ G_2 x_0 (t_3 - t_2) + \frac{G_2 v_x}{2}(t_3^2 - t_2^2)$$

$$- G_3 x_0 (t - t_3) - G_3 v_x (t^2 - t_3^2)\Big]$$

$\boxed{t_1 = 0,\ t_2 = \tau, t_3 = 3\tau}$ — time origin shift

$$= \gamma\left[-G_1 x_0 \tau - \frac{G_1 V_x}{2}\tau^2 \right.$$
$$+ G_2 x_0 2\tau + \frac{G_2 V_x}{2} 8\tau^2$$
$$\left. - G_3 x_0 (t - 3\tau) - \frac{G_3 V_x}{2}(t^2 - 9\tau^2) \right]$$

time origin shift relative to TE $\Rightarrow \underline{4\tau}$ $t' = t - 4\tau$

$$= \gamma\left[-G_1 x_0 \tau - \frac{G_1 V_x}{2}\tau^2 \right.$$
$$+ G_2 x_0 2\tau + \frac{G_2 V_x}{2} 8\tau^2$$
$$\left. - G_3 x_0 (t' + 4\tau - 3\tau) - \frac{G_3 V_x}{2}(t'^2 + 16\tau^2 + 8t'\tau - 9\tau^2) \right]$$

$$\phi = \gamma \Big[-G_1 x_0 \tau - \frac{G_1 V_x}{2} \tau^2$$

$$+ G_2 x_0 2\tau + \frac{G_2 V_x}{2} 8\tau^2$$

$$- G_3 x_0 \tau - \frac{G_3 V_x}{2} 16\tau^2 + \frac{G_3 V_x}{2} 9\tau^2$$

$$- G_3 x_0 t' - \frac{G_3 V_x}{2} t'^2 - \frac{G_3 V_x}{2} 8 t' \tau \Big]$$

For stationary spins, $V_x = 0$

For echo to happen, $\underline{\underline{\phi = 0}}$ @ $t' = 0$

$\therefore \ \phi = 0 = \gamma x_0 \tau [-G_1 + 2G_2 - G_3]$

$\Rightarrow \quad \underline{2G_2 = G_1 + G_3}$ — Condition 1

For moving spins, again, for the echo to occur @ $t' = 0$, $\underline{\underline{\phi_v = 0}}$

$$\phi_v = 0 = -\frac{G_1 v_x \tau^2}{2} + \frac{G_2 v_x 8\tau^2}{2} - \frac{G_3 v_x 16\tau^2}{2} + \frac{G_3 v_x}{2} 9\tau^2$$

$$\Rightarrow \quad 8G_2 - G_1 - 7G_3 = 0$$

$$8G_2 = G_1 + 7G_3$$

Substituting condition (1).

$$4G_1 + 4G_3 = G_1 + 7G_3$$

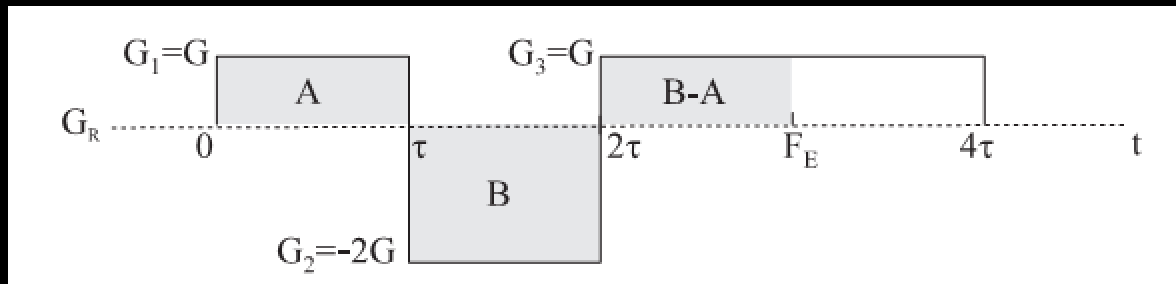
$$G_1 = G_3$$

$$\therefore 2G_2 = G_1 + G_3$$

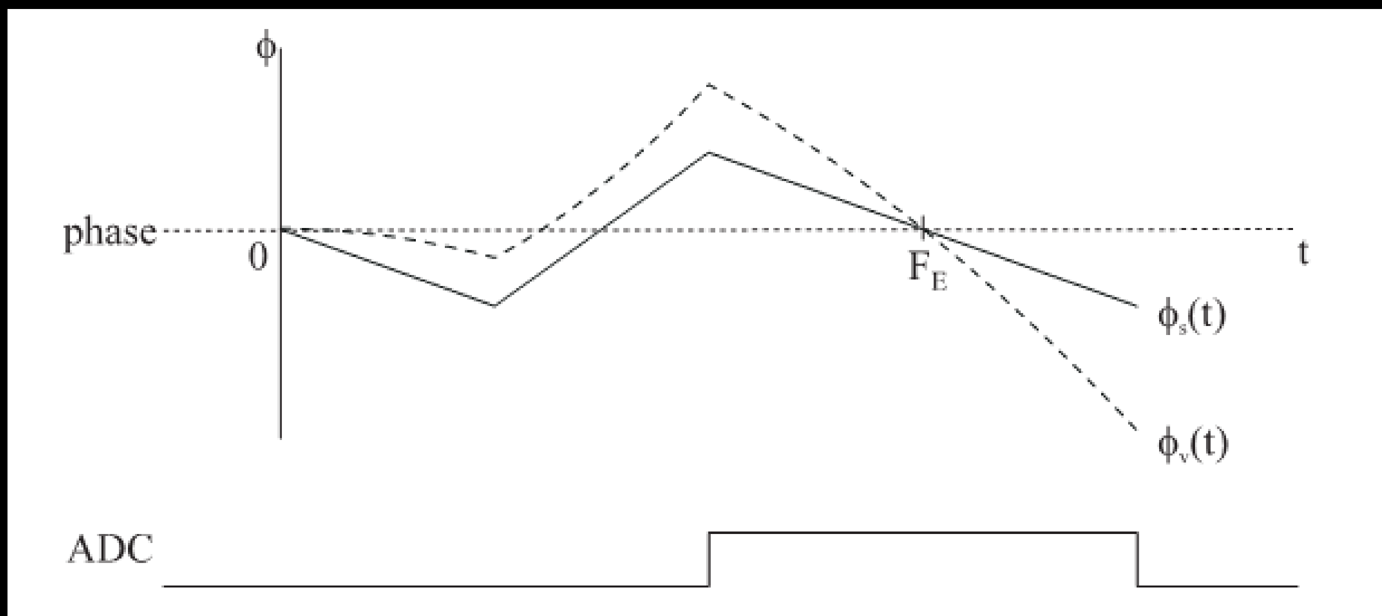
$$\Rightarrow \underline{\underline{G_1 = G_3 = G_2}}$$

$$\phi_x(t) = -\gamma G x_0 t' - \frac{G v_x}{2} t'^2 - \frac{G v_x}{2} 8 t' \tau$$

$$= -\gamma G t' \left[x_0 + 4 v_x \tau + \frac{v_x}{2} t' \right]$$



G_1=G
A
G_3=G
B-A
G_R
0
τ
2τ
F_E
4τ
t
B
G_2=-2G



φ
phase
0
t
F_E
ϕ_s(t)
ϕ_v(t)
ADC

$$G_1\tau + G_2\tau + G\tau = 0$$

i.e.,

$$G_1 + G_2 + G = 0$$

and

$$\frac{1}{2}G_1\tau^2 + \frac{1}{2}G_2(4\tau^2 - \tau^2) + \frac{1}{2}G(9\tau^2 - 4\tau^2) = 0$$

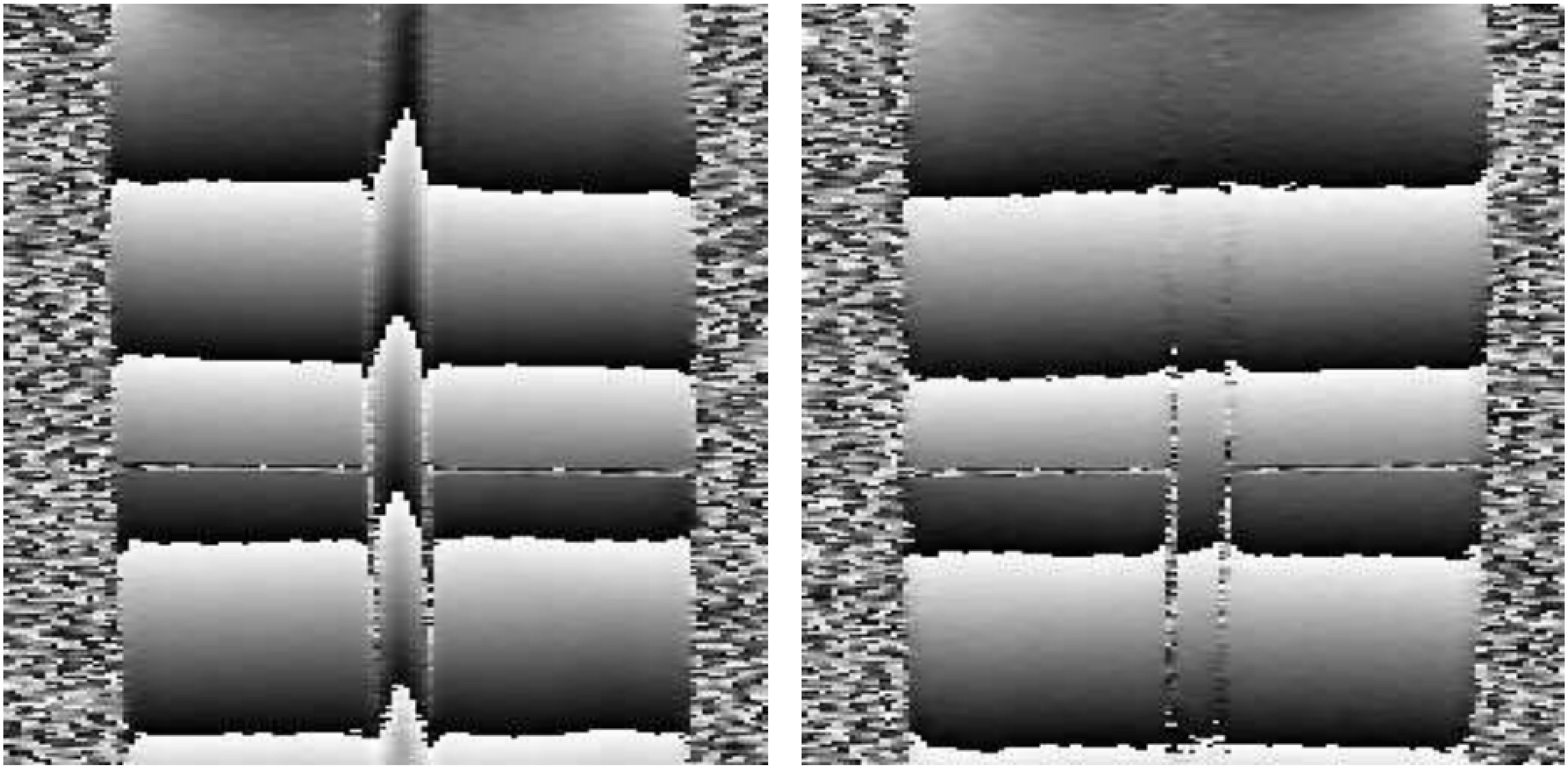
or

$$G_1 + 3G_2 + 5G = 0$$

Solving these equations yields

$$\begin{aligned} G_1 &= G \quad \text{and} \\ G_2 &= -2G \end{aligned}$$

[redacted]



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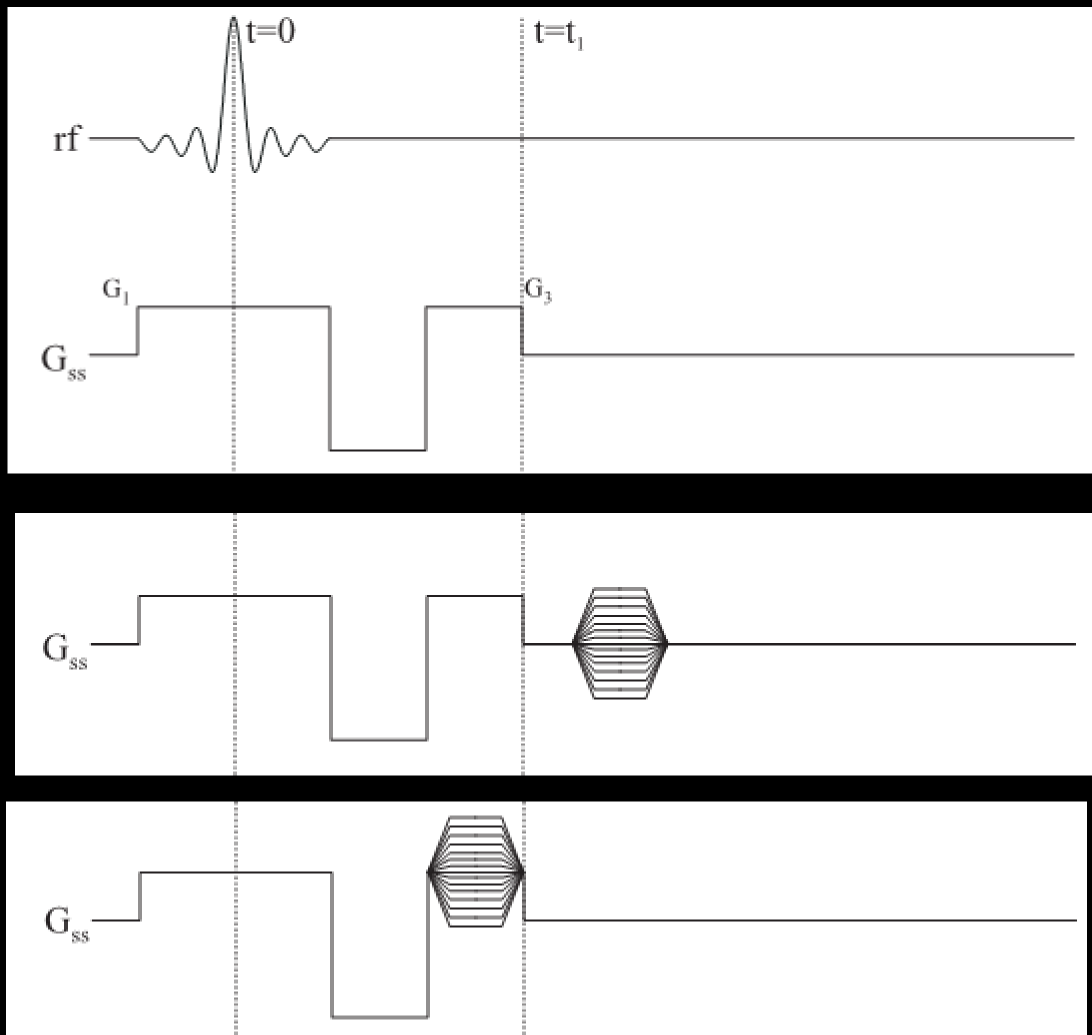
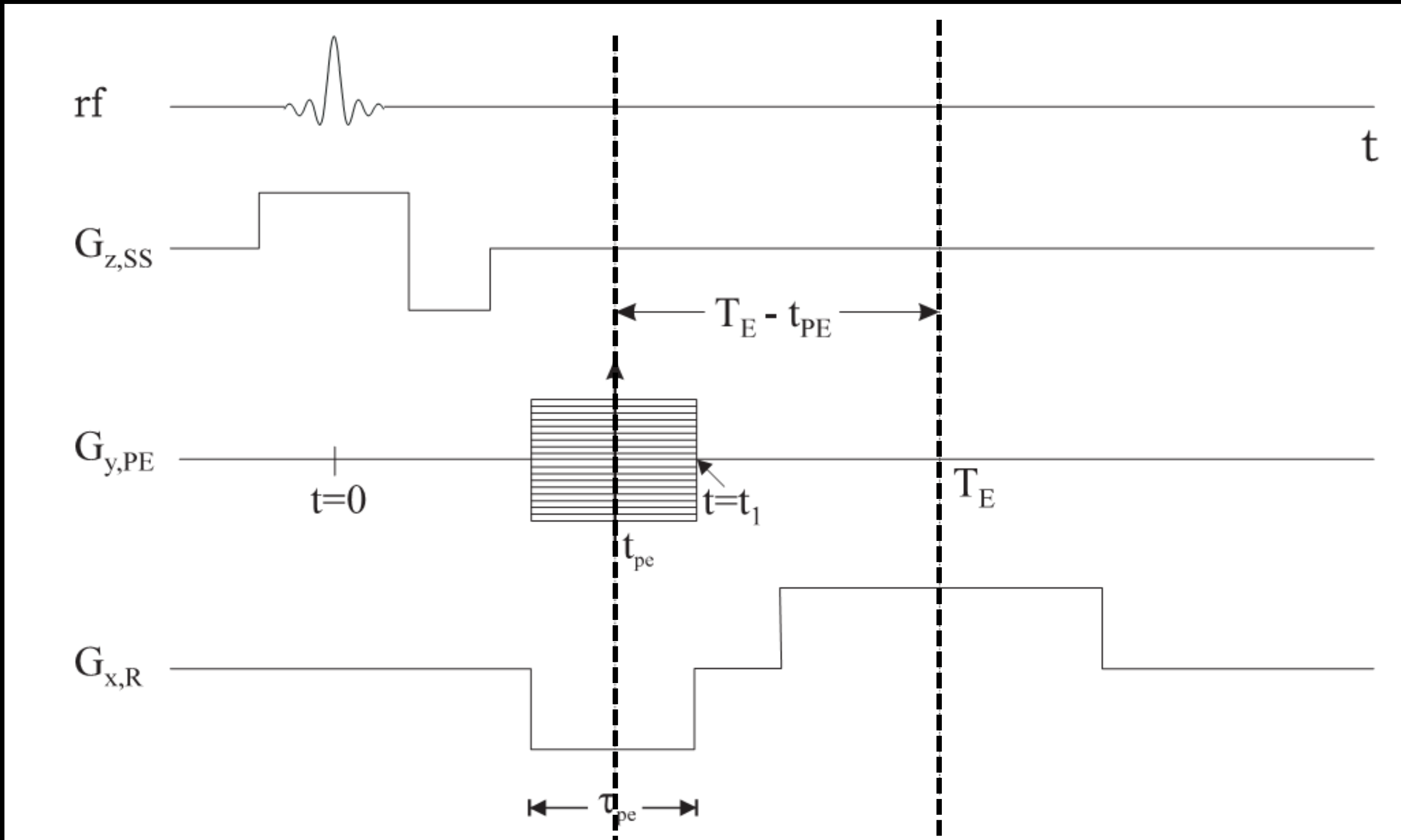
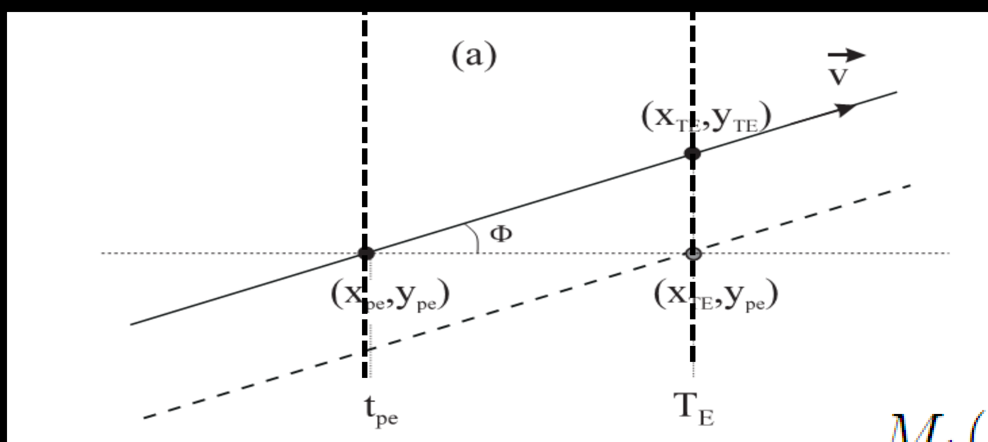


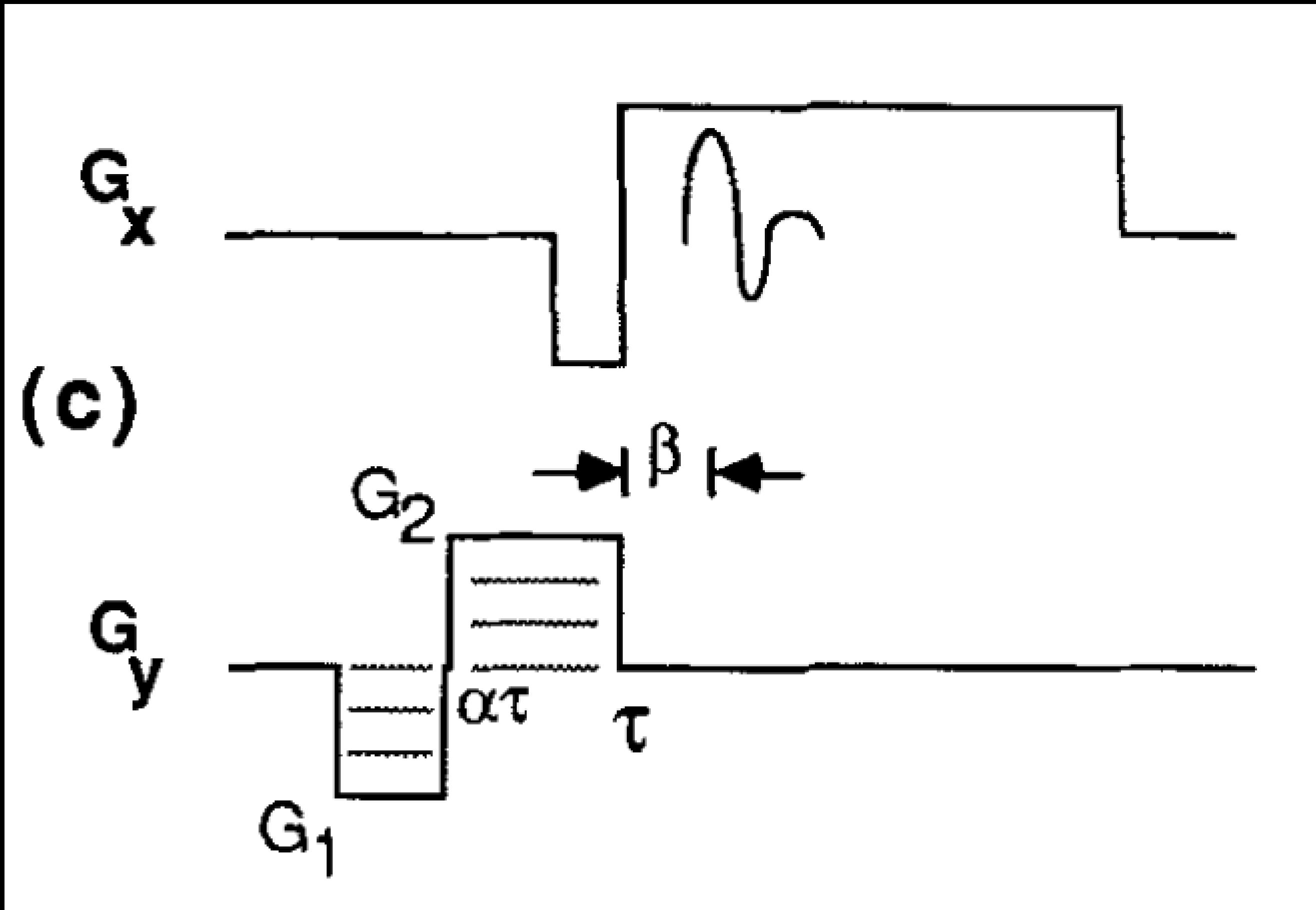
Flow Compensation in Slice Select Direction





Phase encoding Direction

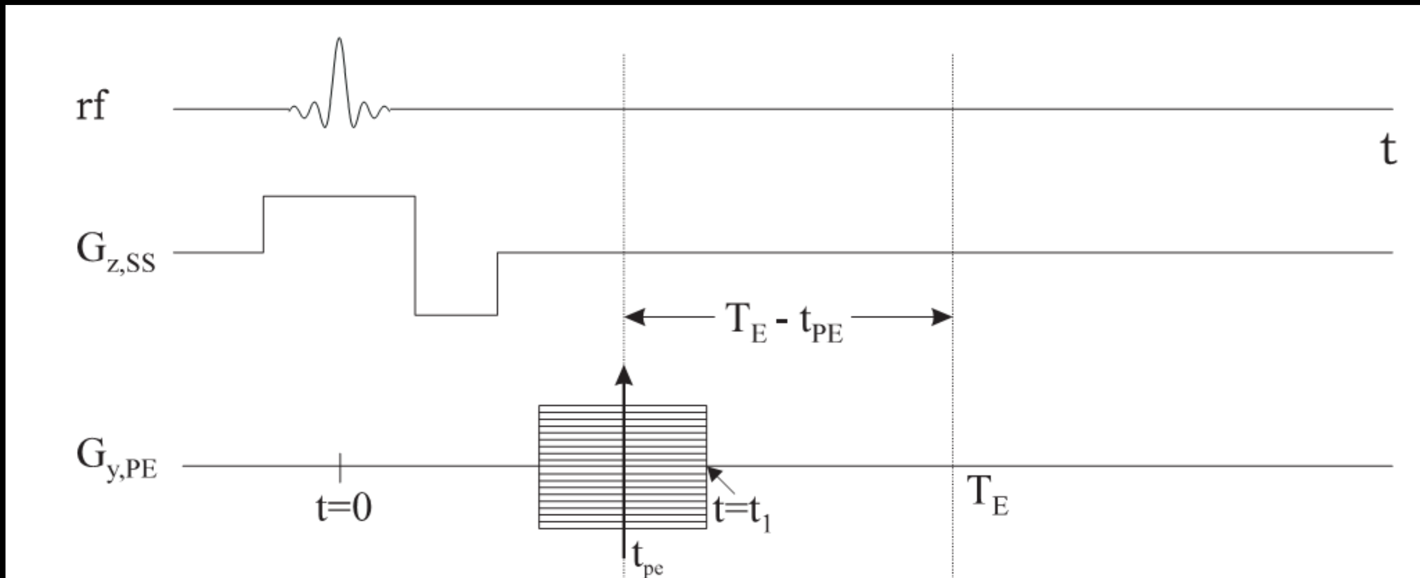
$$M_1(t_1) = \frac{G_y}{2}\left(2\tau_{pe}t_1' - \tau_{pe}^2\right)$$



$$\phi = \gamma G_y(y_0 + v_y \mathrm{TE})\tau.$$

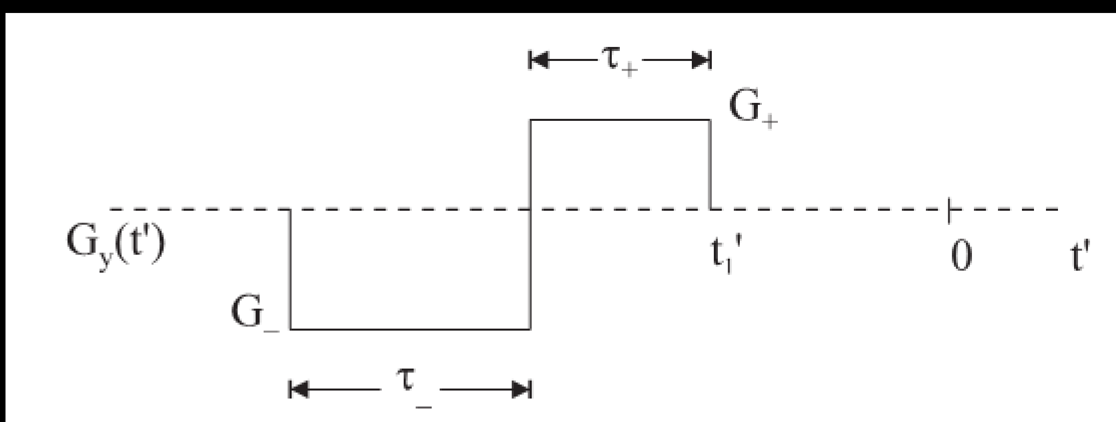
$$G_1\alpha\tau + G_2(1-\alpha)\tau = G_y\tau$$

$$G_1\alpha^2\frac{\tau^2}{2} + G_2(1-\alpha^2)\frac{\tau^2}{2} = G_y\tau\mathrm{TE}$$



$$M_1(t_1) = \frac{G_y}{2}\left(2\tau_{pe}t_1' - \tau_{pe}^2\right)$$

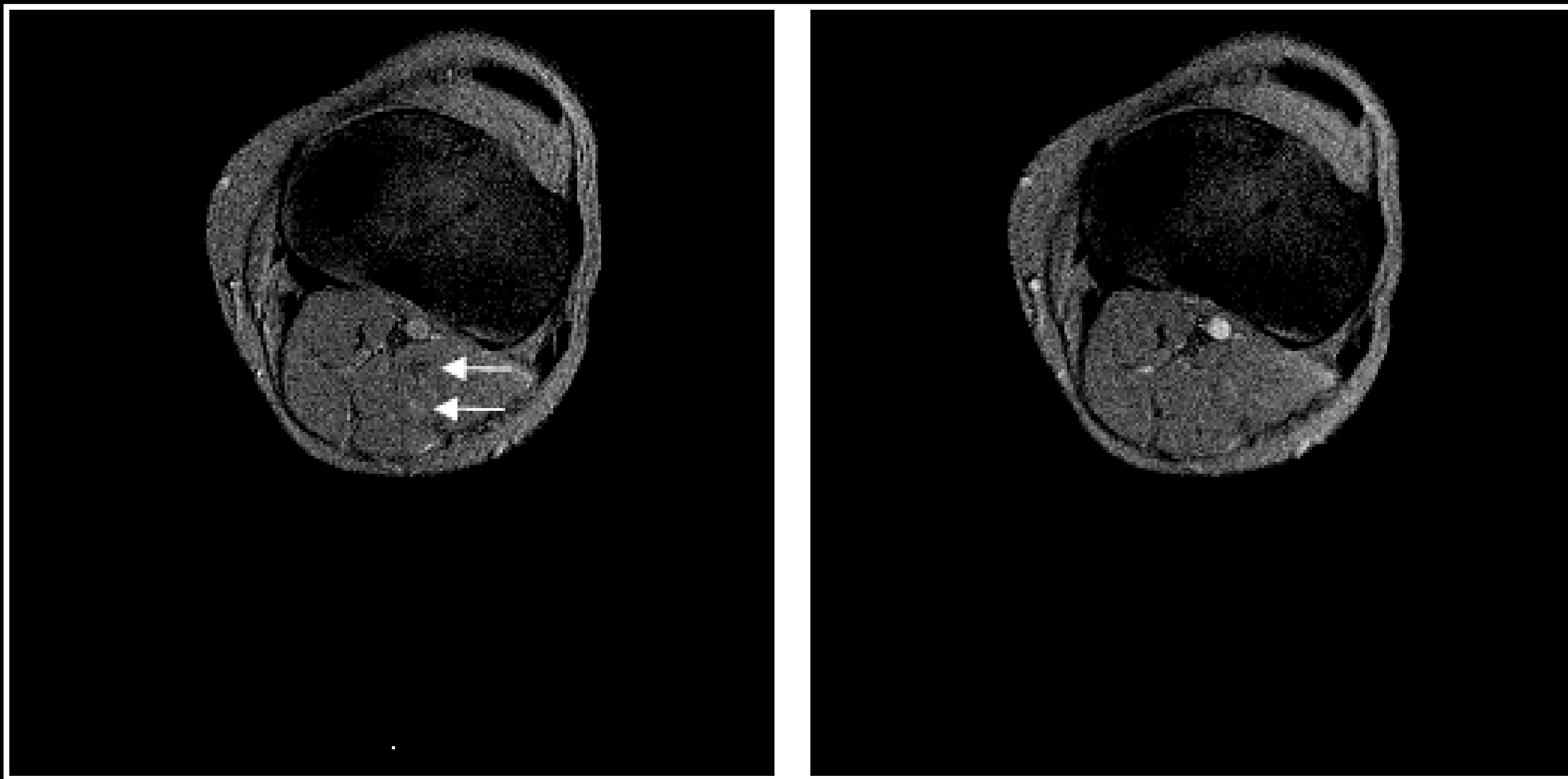
$$\begin{aligned}
\phi_v(t_1) &= -\gamma v_y G_y \tau_{pe}\left(t_1' - \frac{\tau_{pe}}{2}\right) \\
&= -2\pi k_y\left[v_y\left(t_1' - \frac{\tau_{pe}}{2}\right)\right] \\
&\equiv -2\pi k_y \Delta y_v
\end{aligned}$$



$$M_0(0) \equiv \int_{-\infty}^{0} \Delta G_y(t)dt = \frac{1}{\gamma L_y}$$

$$M_1(0) \equiv \int_{-\infty}^{0} t\Delta G_y(t)dt = 0$$

Examples



Uncompensated

signal loss and Ghosting (due to pulsatile flow)

Compensated

$$x(t') = x \quad + \alpha \sin(\omega_m t')$$

$$\begin{aligned} t' &= (G_y/\Delta G + n_y)T_R \\ &= (k_y/\Delta k_y + n_y)T_R \end{aligned}$$

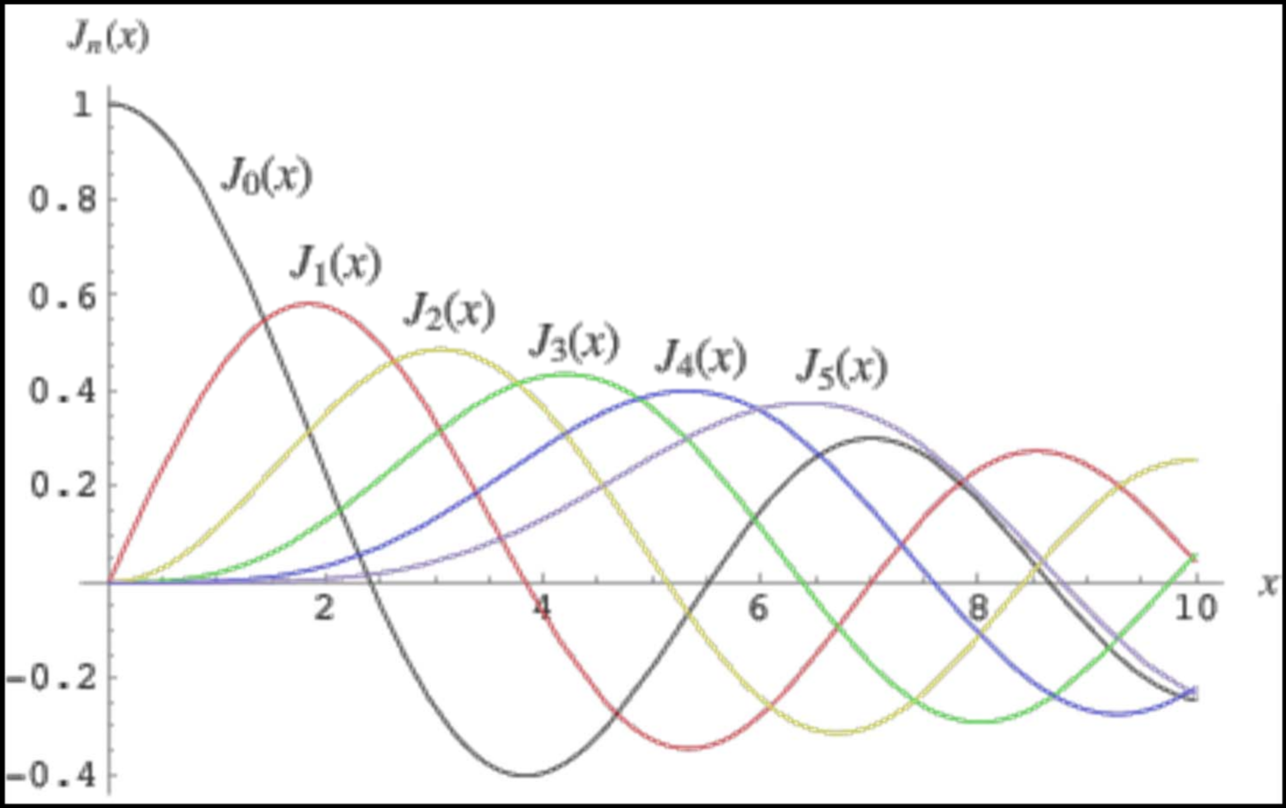
$$s(k_x, k_y) = \int \int dx\, dy\, \rho(x(t'), y) e^{-i2\pi(k_x x(t') + k_y y)}$$

$$x' = x - \alpha \sin \omega_m t'$$

$$s(k_x, k_y) = \int \int dx'\, dy\, \rho(x', y) e^{-i2\pi k_x (x' + \alpha \sin(\omega_m t'))} e^{-i2\pi k_y y}$$

$$j_0(x) = \frac{\sin x}{x}$$
$$j_1(x) = \frac{\sin x}{x^2} - \frac{\cos x}{x}$$
$$j_2(x) = \left(\frac{3}{x^2} - 1\right)\frac{\sin x}{x} - \frac{3\cos x}{x^2}$$
$$j_3(x) = \left(\frac{15}{x^3} - \frac{6}{x}\right)\frac{\sin x}{x} - \left(\frac{15}{x^2} - 1\right)\frac{\cos x}{x},$$

$$\exp\left\{-(ia\sin\omega_m t')\right\} = \sum_{p=-\infty}^{\infty} J_p(a)e^{-ip\omega_m t'}$$



position changing every TR in a periodic manner.

$$x(TR) = x' + \alpha \sin\left(\omega\left(\frac{k_y}{\Delta k_y}\right) TR\right)$$

$$S(k_x, k_y) = \iint dx\,dy\; \rho(x(t'), y)\; e^{-j 2\pi k_x \left(x' + \alpha \sin\left(\omega \frac{k_y}{\Delta k_y} TR\right)\right)}\; e^{-j 2\pi k_y y}$$

$$S(k_x, k_y) = \iint dx'\,dy\; \rho(x', y)\; e^{-j 2\pi k_x x'}\; e^{-j 2\pi k_y y}\; e^{-j 2\pi k_x \alpha \sin\left(\omega \frac{k_y}{\Delta k_y} TR\right)}$$

with $\underline{\underline{A = 2\pi k_x \alpha}}$ $\qquad e^{-iA \sin\left(\omega \frac{k_y}{\Delta k_y} TR\right)} = \sum_{p=-\infty}^{\infty} J_p(A)\, e^{-p\omega \frac{k_y TR}{\Delta k_y}}$

$$S(k_x, k_y) = \sum_{p=-\infty}^{\infty} \iint dx'\,dy\; \rho(x', y)\; J_p(2\pi k_x \alpha)\; e^{-j 2\pi (k_x x' + k_y y)}\; e^{-jp \frac{\omega k_y}{\Delta k_y} TR}$$

$$\Delta y(p) = \left[p\left(\frac{T_R}{T}\right)L_y\right] \text{modulo}\, L_y$$

Spin densities changing every TR

$$\rho(x,y,TR) = \rho_0(x,y) + \alpha \sin\left(\frac{\omega k_y}{\Delta k_y} TR\right)$$

$$S(k_x,k_y) = \iint dx\,dy \left[\rho_0(x,y) + \alpha \sin\left(\frac{\omega k_y}{\Delta k_y} TR\right)\right] e^{-j2\pi(k_x x + k_y y)}$$

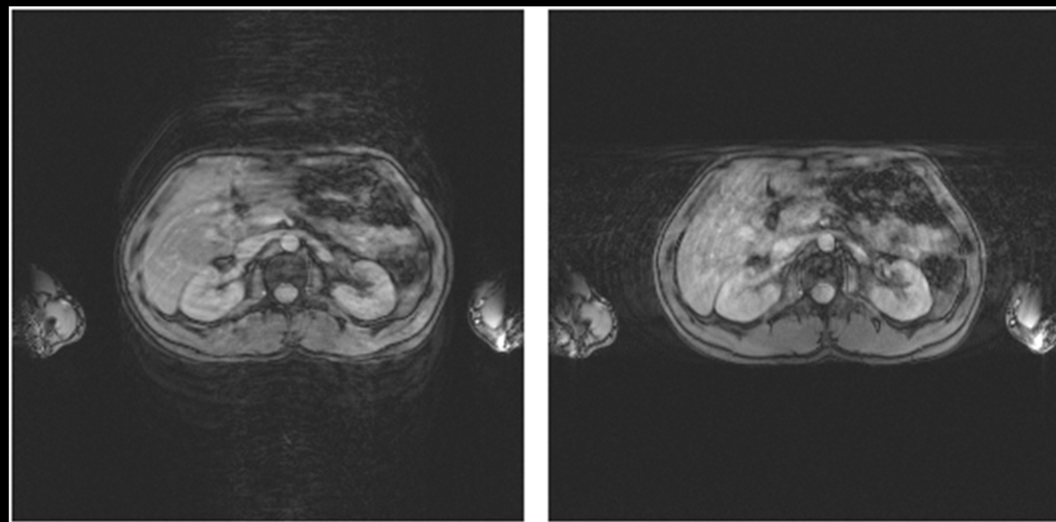
$$S(k_x,k_y) = \alpha \sin\left(\frac{\omega k_y}{\Delta k_y} TR\right) \iint dx\,dy\, e^{-j2\pi(k_x x + k_y y)}$$

$$+ \iint dx\,dy\, \rho_\alpha(x,y)\, e^{-j2\pi(k_x x + k_y y)}$$

Respiratory motion

- Assume $x(t') = x_0 + \alpha \sin(\omega t')$, where $\omega = 2\pi/T$. T is the period of the motion.
- $t' = (G_y/\Delta G + n_y)\, T_R = (k_y/\Delta k_y + n_y)\, T_R$
- Thus, the ghosting due to the respiratory motion only occurs along the phase encoding direction.
- Try to choose T_R to be multiples of T.
- However, this may not be possible. Thus, choose $n_{acq}T_R$ to be roughly T and average the images over n_{acq}. Blurring will occur.

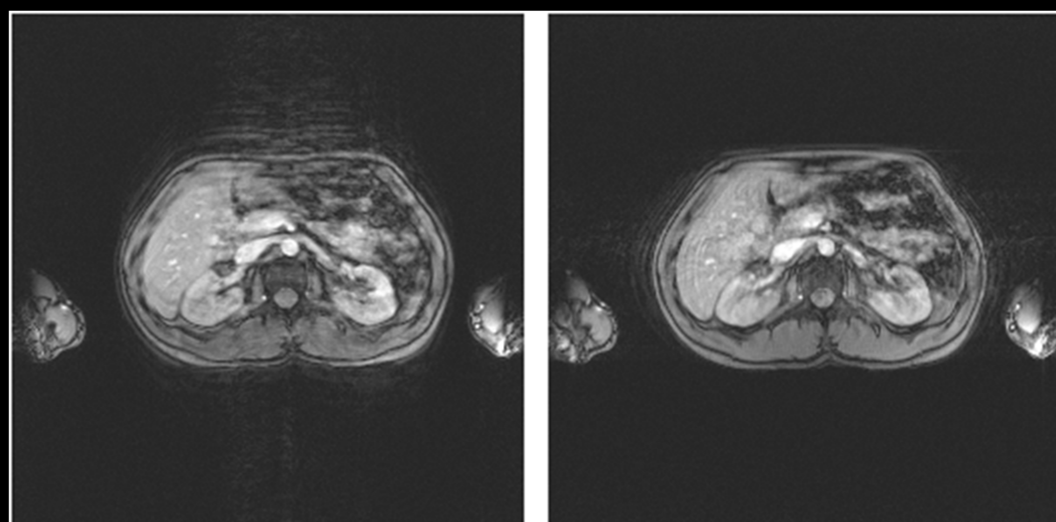
Examples: minimizing ghosting



T_R = 400 ms

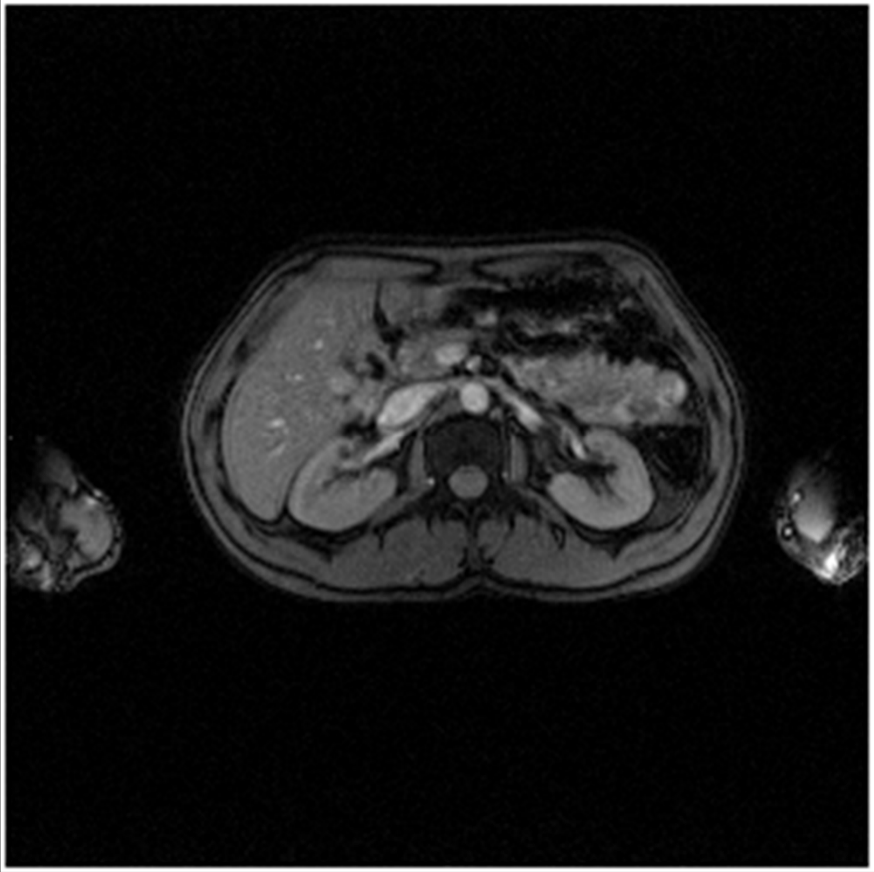
Phase encoding ↑

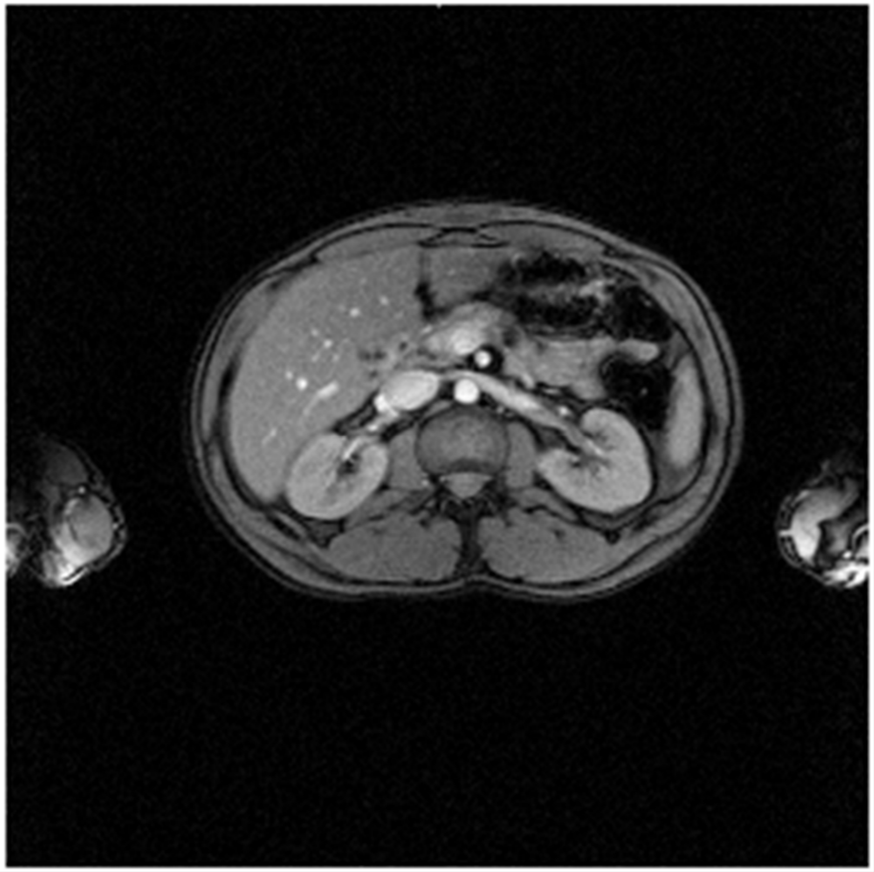
Read direction ↑



T_R = 200 ms

Examples: eliminating ghosting



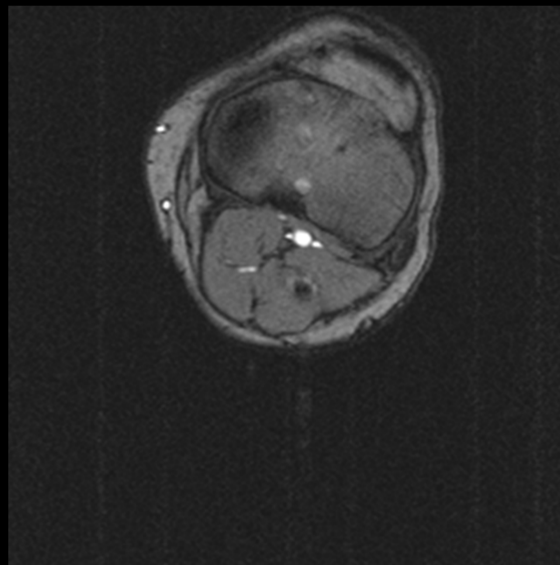


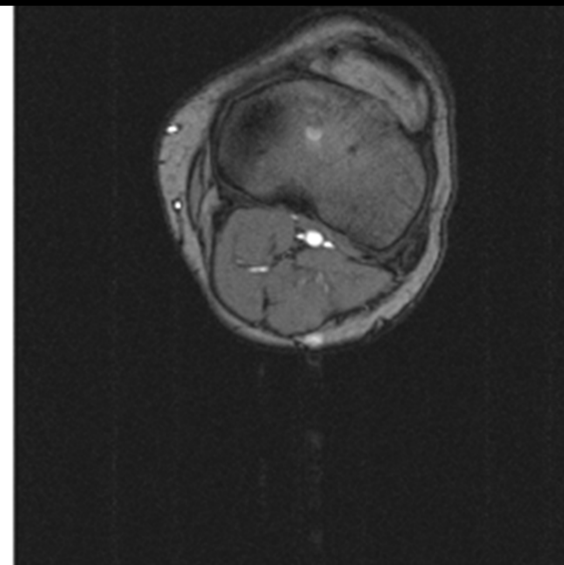
T_R = 50 ms, T ~ 4 sec.

Breath-hold

Examples: pulsatile flow

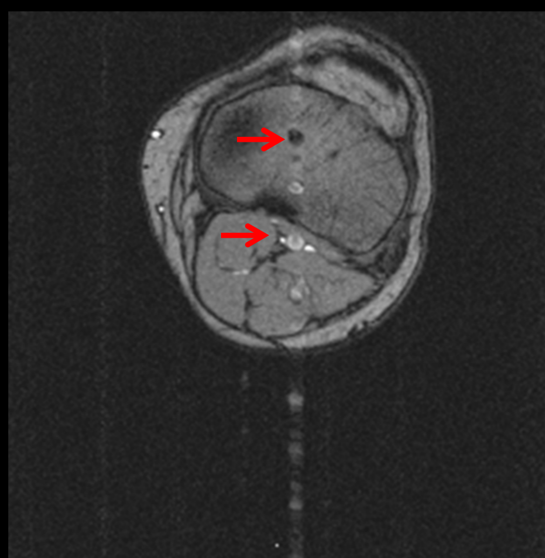
T_R = 14 ms,
N_{acq} = 1.





T_R = 14 ms,
N_{acq} = 2.

similar!



T_R = 14 ms,
N_{acq} = 1.
uncompensated

T_R = 28 ms,
N_{acq} = 1.